## Proportion *z*-Tests

In the previous sections, we were interested in doing inference on $\mu$, the population mean. In this section, we are concerned with doing inference on $\rho$ (rho), the population proportion or percentage. The following situations are all one-proportion situations.

- we wish to predict the percentage of the population who will vote for a particular candidate by polling a sample of the population.
- we wish to predict the percentage of people who think the coach of a team should be fired by polling a sample of the fan base.
- It is thought that 60% of drivers would drive a mile out of their way to save 5 cents a gallon on gas. A sample of drivers is polled and an inference test is performed on the 60% figure.
- A popcorn company claims that no more than 5% of the kernels will remain un-popped when put in a microwave. A sample of popcorn packages were tested against the claim of 5%.

**Given information**:
$n$ – the number of trials in the sample $\hat{p}$ - the percentage of successes in the sample
**Parameter of interest**: $\rho$ - the true population percentage of success

**Conditions for inference about a proportion:**

- The data is from an SRS from the population of interest
- The population is at least 10 times as large as the sample
- In a confidence interval, $n$ is so large that $n\hat{p}$ and $n(1-\hat{p})$ are 10 or more
- In a test of $H_0 : \rho = \rho_0$, $n\rho_0$ and $n(1-\rho_0)$ are both 10 or more

**Null and Alternative Hypotheses:** $H_0 : \rho = \rho_0$ $H_a : \rho > \rho_0$ or $\rho < \rho_0$ or $\rho \neq \rho_0$

**Appropriate statistic:**

Proportion tests are *z*-tests. The *z* statistic is: $z = \dfrac{\hat{\rho} - \rho_0}{\sqrt{\dfrac{\rho_0(1-\rho_0)}{n}}}$

For confidence intervals for $\rho$, there is no specific value to substitute. We use the fact that in large samples, $\hat{\rho}$ will be close to $\rho$ so we replace the standard deviation by the standard error of $\hat{\rho}$.

SE = $\sqrt{\dfrac{\hat{\rho}(1-\hat{\rho})}{n}}$ and the confidence interval is in the form: $\hat{\rho} \pm z^* \sqrt{\dfrac{\hat{\rho}(1-\hat{\rho})}{n}}$

***p-value***

Since the proportion tests of this chapter use the *z*-statistic, you use table A (standard normal probabilities) to calculate the *p-value*. Later, you will use the calculator to find the *p-value*.

**Choosing the sample size:**

To determine the sample size which will yield a level C confidence for a population proportion *p* with a specified margin of error less than or equal to *m* and solve for *n*.

$z^* \sqrt{\dfrac{\rho^*(1-\rho^*)}{n}}$ where *p** is a guessed value for the sample proportion. The most conservative choice of *p** is 0.5.

Example 1) Are pennies weighted exactly the same on each side? We will test this hypothesis by our experiment in the cafeteria. Carefully balance 10 coins on their sides, slap the table, and write down how many of them landed heads up. Write down the results of the experiment for the entire class.

| Trial | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Heads up | | | | | | | | | | | | | | | | | | | | | |

Here is data for those unable to do the problem.

| Trials | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Heads up | 4 | 3 | 6 | 5 | 5 | 4 | 4 | 5 | 3 | 4 | 3 | 3 | 5 | 7 | 3 | 2 | 5 | 4 | 5 | 4 | 3 |

Test the hypothesis at the 95% level that pennies are equally weighted on either side. Then find and interpret a 95% confidence interval for the data.

**Test:**

**Given information:**

**Parameter of interest (appropriate to the problem situation):**

**Conditions:**

**Null and Alternative Hypotheses:**

**Appropriate statistic (either by shown formula or calculator – underline)**

***p*-value ___________________**

**Conclusion regarding Ho: ___________________________________________**

**Conclusion for problem (in clear English):**

**Confidence interval (either by shown formula or calculator – underline) _____________________**

**Meaning of confidence interval in words**

How many pennies would we need to sample to estimate the percentage of heads up with 95% and margin of error no greater than 1%?